Exercise sheet 12

1. Let \( \mathcal{O}_K \) be the ring of integers in a number field \( K \). Show that there is a canonical isomorphism between the group of Weil divisors modulo linear equivalence, and the ideal class group of \( \mathcal{O}_K \).

2. Let \( A \) be a noetherian ring and \( p \) a minimal prime ideal. Show that \([V(p)]\) is nonzero in \( \text{CH}_*(A) \).

3. Let \( A \) be an integral domain. Show that the set \( \text{Cart}^+(A) \) of effective Cartier divisors admits a canonical monoid structure, and there is a canonical injective homomorphism

\[
\text{Cart}^+(A) \to \text{Cart}(A)
\]

which exhibits \( \text{Cart}(A) \) as the group completion of \( \text{Cart}^+(A) \).

4. Let \( A \) be a noetherian ring of dimension \( d \). Recall the homomorphism \( \gamma : Z_*(A) \to G_0(A) \) defined in Sheet 9, Exercise 3. Note that \( \gamma \) sends \( Z_k(A) \) to \( G_0(A)_{\leq k} \), the subgroup generated by classes \([M]\) such that \( \dim(\text{Supp}_A(M)) \leq k \).

Let \( M \in \text{Mod}^f_A \) and suppose that \( \text{Supp}_A(M) \) is of pure dimension \( n \). Prove the formula

\[
\gamma([M]_n) = [M]
\]

in \( G_0(A)_{\leq n}/G_0(A)_{\leq n-1} \).