Exercise sheet 12

1. Let $\mathcal{O}_K$ be the ring of integers in a number field K. Show that there is a canonical isomorphism between the group of Weil divisors modulo linear equivalence, and the ideal class group of $\mathcal{O}_K$.

2. Let A be a noetherian ring and $\mathfrak{p}$ a minimal prime ideal. Show that $[V(\mathfrak{p})]$ is nonzero in $CH_*(A)$.

3. Let A be an integral domain. Show that the set $\mathrm{Cart}^+(A)$ of effective Cartier divisors admits a canonical monoid structure, and there is a canonical injective homomorphism

$$\mathrm{Cart}^+(A) \to \mathrm{Cart}(A)$$

which exhibits $\mathrm{Cart}(A)$ as the group completion of $\mathrm{Cart}^+(A)$.

4. Let A be a noetherian ring of dimension d. Recall the homomorphism $\gamma : Z_*(A) \to G_0(A)$ defined in Sheet 9, Exercise 3. Note that γ sends $Z_k(A)$ to $G_0(A)_{\leqslant k}$, the subgroup generated by classes [M] such that $\dim(\mathrm{Supp}_A(M)) \leqslant k$.

Let $M \in \mathrm{Mod}_A^{\mathrm{fg}}$ and suppose that $\mathrm{Supp}_A(M)$ is of pure dimension n. Prove the formula

$$\gamma([M]_n) = [M]$$

in $G_0(A)_{\leqslant n} / G_0(A)_{\leqslant n-1}$.