1. Let $A$ be a DVR with uniformizing parameter $\pi$ (i.e., $\pi$ is a generator of the maximal ideal).
   (a) Show that $A$ is of dimension 1.
   (b) Deduce that $\dim(A[T]) \geq 2$.\(^1\)
   (c) Show that the ideal of $A[T]$ generated by the element $f = \pi T - 1 \in A[T]$ is maximal.
   (d) Let $Y = V((f)) \subset |\Spec(A[T])|$. Show that $\dim(A[T]) \neq \dim(Y) + \codim_{A[T]}(Y)$.
   (e) For a general noetherian ring $A$ and any closed subset $Y \subseteq |\Spec(A)|$, show that $\dim(Y) + \codim_A(Y) \leq \dim(A)$.

2. Let $A$ be a noetherian ring.
   (a) The codimension of any integral closed subset $V(p) \subset |\Spec(A)|$ is given by $\codim_A(V(p)) = \dim(A_p)$.
   (b) Show that the dimension of $A$ is given by the formula $\dim(A) = \sup_x \codim_A(\{x\})$, where the supremum is taken over all closed points $x$ of $|\Spec(A)|$.

3. Let $A$ be a noetherian ring. Define a homomorphism $\gamma_A : Z_*(A) \to G_0(A)$ by sending the class of an integral subset $V(p) \subset |\Spec(A)|$ to the class $[A/p]$.
   (a) Let $k$ be an algebraically closed field and $A = k[T, U]$. Show that $\gamma_A$ descends to a homomorphism $\gamma_A : CH_*(A) \to G_0(A)$ which is invertible.
   (b) Let $A$ be any noetherian ring and $\phi : A \to A/I$ a surjective ring homomorphism. Show that the square

$$
\begin{array}{ccc}
Z_*(A/I) & \xrightarrow{\phi_*} & Z_*(A) \\
\gamma_{A/I} & \downarrow & \gamma_A \\
G_0(A/I) & \xrightarrow{\phi_*} & G_0(A)
\end{array}
$$

commutes.

\(^1\)In fact, one has $\dim(A[T]) = \dim(A) + 1$ for any noetherian ring $A$, but this is non-trivial; see e.g. [Bourbaki, Comm. alg., §3, no. 4, Cor. 3 to Prop. 7].
4. Let $A$ be a noetherian ring and let $V(p)$ and $V(q)$ be distinct integral closed subsets of $|\text{Spec}(A)|$, both of dimension $d$. Prove the formula

$$[A/(p \cap q)]_d = [V(p)] + [V(q)]$$

in $\text{CH}_d(A)$. 