Exercise sheet 10

The minimum passing average is 20 points per sheet.

1. *10 points. Proper base change for nearby cycles.* Let $t : X \to \mathbf{A}^1$ be a regular function on X, $f : X' \to X$ a proper morphism, and $t' : X' \to \mathbf{A}^1$ the induced regular function on X'. Denote by $f_\eta : X'_\eta \to X_\eta$ and $f_0 : X'_0 \to X_0$ the induced morphisms on the generic and special fibres, respectively. Show that there is a canonical isomorphism $\psi_t \circ f_{\eta,*} \simeq f_{0,*} \circ \psi_{t'}$.

2. *10 points. Smooth base change for nearby cycles.* Let the notation be as in the previous exercise, except that $f : X' \to X$ is now a smooth morphism. Show that there is a canonical isomorphism $f_0^* \psi_t \simeq \psi_{t'} \circ f_\eta^*$.

3. *10 points.* Given a unit $\lambda \in R^\times$, let $\mathcal{L}_\lambda \in \mathrm{Loc}(\mathbf{C}^*; R)$ be the locally constant sheaf of finite type whose stalks are isomorphic to R, and whose monodromy is given by the multiplication by λ map $\lambda : R \xrightarrow{\sim} R$. Show that if $R = k$ is an algebraically closed field, then there is a canonical eigenspace decomposition of the exponential sheaf $\mathcal{L}^{\exp} \in \mathrm{Loc}^\diamond(\mathbf{C}^*)$ as follows:
$$\mathcal{L}^{\exp} \simeq \bigoplus_{\lambda \in R^\times} \mathcal{L}_\lambda.$$