Exercise sheet 11

The minimum passing average is 20 points per sheet.

1. *5 points.* Let X be a topological space and $t : X \to \mathbf{C}$ the zero function. Compute the functors ψ_t and ϕ_t in this case.

2. *5 points.* Let $K \in \mathbf{D}(R)$ be a complex and denote by $\mathcal{F} := i_{0,*}(\underline{K}) \in \mathrm{Shv}(\mathbf{C})$ the skyscraper sheaf at $0 \in \mathbf{C}$. Compute $\psi'_{\mathrm{id}}(\mathcal{F}) \simeq \psi_{\mathrm{id}}(\mathcal{F}|_{\mathbf{C}^*})$ and $\phi_{\mathrm{id}}(\mathcal{F})$ for the identity function $\mathrm{id} : \mathbf{C} \to \mathbf{C}$.

3. *5 points.* Give a counterexample to show that the vanishing cycles functor need not commute with j_* for open immersions $j : U \hookrightarrow X$ in $\mathrm{Sch}^{\mathrm{ft}}_{\mathbf{C}}$.

4. *10 points.* Consider the function $t = z_1 z_2$ on $\mathbf{A}^2$. Compute $\phi_t(\underline{R})$.

5. *20 points.* Let X be a smooth algebraic curve, i.e., a smooth $\mathbf{C}$-scheme of finite type of dimension 1. Let $\mathcal{F} \in \mathrm{Shv}_c(X)$ be a constructible sheaf on X. Show that $\mathcal{F}$ is a local system if and only if for every point $x \in X$ the stalk $x^*\phi_t(\mathcal{F}) \simeq 0$ vanishes, where t is a local coordinate[1] at x. (For a hint on sufficiency, see [Dim, p. 110].)

References

[Dim] A. Dimca, *Sheaves in Topology.*

[1] i.e., t is a uniformizer of the discrete valuation ring $\mathcal{O}_{X,x}$. To spell this out: $t : U \to \mathbf{A}^1$ is a regular function on a Zariski open neighbourhood $U \ni x$ such that $t(x) = 0$ and $dt_x \neq 0$ in the cotangent space $\mathfrak{m}_x/\mathfrak{m}_x^2$.