Exercise sheet 7

The minimum passing average is 20 points per sheet.

1. *5 points.* Show that $\mathrm{C}_\bullet^{\mathrm{BM}}(\mathbf{R}^n; R) \simeq \mathrm{C}^\bullet(\mathbf{R}^n; R)[n]$ for any $n \geqslant 0$. In particular $\mathrm{H}_i^{\mathrm{BM}}(\mathbf{R}^n; R) \simeq \mathrm{H}^{n-i}(\mathbf{R}^n; R)$.

2. *10 points.* Let $I = [0,1]$ be the closed unit interval, $I^\circ = (0,1)$, and $\partial I = I \backslash I^\circ = \{0,1\}$. Let $i : \partial I \hookrightarrow I$ and $j : I^\circ \hookrightarrow I$ be the inclusions. Show the following identities:
   (1) $\omega_{I^\circ} \simeq \underline{R}_{I^\circ}[1]$.
   (2) $\omega_{\partial I} \simeq \underline{R}_{\partial I}$.
   (3) $\underline{R}_I \simeq j_*(\underline{R}_{I^\circ})$.
   (4) $\omega_I \simeq j_!(\underline{R}_{I^\circ})[1]$.

3. *20 points.* Show that $a^* : \mathrm{Loc}^\diamond(\mathrm{pt}) \to \mathrm{Loc}^\diamond([0,1])$ is an equivalence, where $a : [0,1] \to \mathrm{pt}$ is the projection, by reducing to the case of discrete locally constant sheaves considered in Sheet 6. (Note that the claim is equivalent to the assertion that every locally constant sheaf on $[0,1]$ is constant.)