Exercise sheet 0

1. Show that $\mathbf{Z}/n\mathbf{Z}$ is not projective as a $\mathbf{Z}$-module.

2. Let $\phi : \mathrm{M} \to \mathrm{N}$ be a homomorphism of f.g. projective A-modules. Show that neither the kernel nor the cokernel is necessarily projective.

3. Let A be a commutative ring and $f \in \mathrm{A}$ an element. Show that there is a canonical isomorphism of rings $\mathrm{A}[f^{-1}] \to \mathrm{A}[f^{-1}] \otimes_{\mathrm{A}} \mathrm{A}[f^{-1}]$, and moreover that $\mathrm{Tor}_i^{\mathrm{A}}(\mathrm{A}[f^{-1}], \mathrm{A}[f^{-1}]) = 0$ for all $i > 0$.

4. Let A be a commutative ring and $\mathrm{I} \subseteq \mathrm{A}$ an ideal. Show that there is a canonical isomorphism of rings $\mathrm{A}/\mathrm{I} \to \mathrm{A}/\mathrm{I} \otimes_{\mathrm{A}} \mathrm{A}/\mathrm{I}$, but that $\mathrm{Tor}_1^{\mathrm{A}}(\mathrm{A}/\mathrm{I}, \mathrm{A}/\mathrm{I})$ may not vanish.